Physics IV
ISI B.Math
Semestral Exam : April 24,2017

Total Marks: 50
Time : 3 hours
Answer all questions

1. (**Marks: 3 + 7 = 10**)

(a) Show that the sum of two future pointing timelike four vectors is future pointing timelike.

(b) A particle with mass m and energy E approaches an identical particle at rest. They collide elastically in a way such that both particles scatter at an angle θ relative to the incident direction. Find θ in terms of E and m. What is θ in the extreme relativistic and non relativistic limits?

2. (**Marks : 5 + 5 = 10**)

(a) A train with proper length L moves at speed $\frac{5c}{13}$ with respect to the ground. A ball is thrown from the back of the train to the front. The speed of the ball with respect to the train is $\frac{c}{3}$. As viewed by someone on the ground, how much time does the the ball spend in the air and how far does it travel ?

(b) If $\psi(x,t)$ is a solution of the time dependent Schrödinger equation for a particle of mass m moving in a potential $V(x)$, show that the following "continuity equation" is obeyed by the probability density $\rho = \psi^*\psi$ and the probability current density $J = \frac{i\hbar}{2m}[\psi\frac{\partial\psi^*}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}]$

$$\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0$$

3. (**Marks : 4 + 2 + 2 + 2 = 10**)

A particle mass m moves under the influence of the potential $V(x) = 0$ if $0 \leq x \leq a$ and ∞ otherwise.

(a) Solve the time independent Schrödinger equation for this potential and find the stationary states $\psi_n(x)$ and their corresponding energies E_n.

(b) If the initial state is given by $\Psi(x,0) = \frac{[2\psi_1(x)+\psi_2(x)]}{\sqrt{5}}$,find $|\Psi(x,t)|^2$

(d) If one measured the energy of the particle at any time $t > 0$, what values will one get and what is the probability of getting each of them ?

(e) Find $< H >$, where H is the Hamiltonian operator.

4. (**Marks : 1 + 2 + 2 + 5 = 10**)

A one dimensional harmonic oscillator of mass m has potential energy $V(x) = \frac{1}{2}m\omega^2x^2$. Consider the operators $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$

(a) Do a and $a^\dagger$ correspond to physically measurable observables ? Justify your answer.

(b) Express the Hamiltonian $\hat{H}$ in terms of the number operator $N = a^\dagger a$ Find the eigenvalues of the energy in terms of the eigenvalues n of the number operator where $n = 0, 1, 2 \cdots$

(c) Show that if $|n>$ is an eigenvector of the number operator N with eigenvalue n, then $a^\dagger|n>$ is an eigenvector of N with eigenvalue $n+1$

(d) Find the expectation value of the potential energy $< V >$ in the state $|n>$. Remember that $|n>$s form an orthonormal set. Show that the ground state $|0>$ is a minimum uncertainty state

5. (**Marks : 5 + 5 = 10**)

(a) Consider a free particle of mass m moving in one dimension. Suppose that at $t = 0$ the system is in the state $\psi(x, 0) = \sqrt{\frac{1}{a}}$ for $|x| < \frac{a}{2}$ and $= 0$ elsewhere. Is this a state of definite energy ? If at the same instant, the momentum of the particle is measured, find the probability that a measurement of momentum yields the value $p = \hbar k$ in the interval $\hbar k, \hbar(k + dk)$.

(b)Consider an observable represented by an operator $\hat{A}$ which does not explicitly depend on time and whose commutator with the Hamiltonian $\hat{H}$ is the constant c, $[\hat{H}, \hat{A}] = c$. Find $< \hat{A} >$ at $t > 0$, given that the system is in a normalized eigenstate of $\hat{A}$ at $t = 0$, corresponding to the eigenvalue a.